

SOCIETY OF ACTUARIES
AMERICAN SOCIETY OF PENSION ACTUARIES
JOINT BOARD FOR THE ENROLLMENT OF ACTUARIES

ENROLLED ACTUARIES BASIC EXAMINATION

MAY 2005 EA-1 EXAMINATION

Data for Question 1 (4 points)

A fund has the following transactions for 2005:

| <u>Date</u> | <u>Account Value</u> | <u>Contributions</u> | <u>Benefit Payments</u> |
|-------------|--------------------------|----------------------|-----------------------------|
| 1/1/2005 | \$1,000 | C_1 | 0 |
| 3/31/2005 | - | 0 | \$100 |
| 4/1/2005 | \$1,300 | 0 | 0 |
| 12/31/2005 | - | C_2 | \$150 |
| 1/1/2006 | \$1,700 | 0 | 0 |

The time weighted rate of return in 2005 is 6.25%. The dollar weighted rate of return in 2005 is 6.00%.

Question 1

In what range is C_1 ?

- (A) Less than \$405
- (B) \$405 but less than \$430
- (C) \$430 but less than \$455
- (D) \$455 but less than \$480
- (E) \$480 or more

Data for Question 2 (5 points)

Terms of a serial bond:

| | |
|----------------------|--|
| Face amount of bond: | \$100,000. |
| Terms of redemption: | 10 equal annual installments at 125% of par commencing at end of 10th year after purchase; except that the 4 th redemption is at 100% of par. |
| Coupons: | 6%, payable semi-annually. |
| Yield rate: | 5%, compounded annually. |

Question 2

In what range is the purchase price?

- (A) Less than \$121,850
- (B) \$121,850 but less than \$121,950
- (C) \$121,950 but less than \$122,050
- (D) \$122,050 but less than \$122,150
- (E) \$122,150 or more

Data for Question 3 (3 points)

Terms of a \$1,000 loan issued by Smith:

Length of loan: 20 years.

Payments: Level annual payments at the end of each year.

Interest: 5% nominal, convertible semi-annually.

When Smith receives each payment, it is immediately reinvested at 6%, compounded annually.

R is the effective annual rate of interest earned by Smith on his combined investments over the 20 year period.

Question 3

In what range is R ?

- (A) Less than 5.57%
- (B) 5.57% but less than 5.62%
- (C) 5.62% but less than 5.67%
- (D) 5.67% but less than 5.72%
- (E) 5.72% or more

Data for Question 4 (4 points)

The following annuities immediate are actuarially equivalent:

- I. Straight life annuity of \$100 payable to a life age 65
- II. \$90 payable during the joint life of two independent lives both age 65, reducing to \$54 on the first death
- III. \$70 payable during the joint life of two independent lives both age 65, reducing to P on the first death

Question 4

In what range is P?

- (A) Less than \$57
- (B) \$57 but less than \$59
- (C) \$59 but less than \$61
- (D) \$61 but less than \$63
- (E) \$63 or more

Data for Question 5 (3 points)

Smith (age 65) purchases a single premium annuity:

| | |
|-----------------|---|
| Annual payment: | \$1,000 at the end of each year |
| Payment period: | For Smith's lifetime and continuing for 5 years after Smith's death |
| Interest rate: | 5%, compounded annually |

$$a_{65} = 10.17548$$

Question 5

In what range is the single premium paid by Smith?

- (A) Less than \$12,150
- (B) \$12,150 but less than \$12,250
- (C) \$12,250 but less than \$12,350
- (D) \$12,350 but less than \$12,450
- (E) \$12,450 or more

Data for Question 6 (3 points)

Smith (age 65) purchases an annuity that pays \$1,000 at the end of each year.
Payment ceases at the earlier of Smith's death or 25 years from purchase date.

Interest rate: 5%, compounded annually

Mortality: $p_x = 0.95$, for $x < 75$
Mortality rates at ages 75 and greater are double those under age 75

Y = The present value of this annuity

Question 6

In what range is Y ?

- (A) Less than \$6,500
- (B) \$6,500 but less than \$7,500
- (C) \$7,500 but less than \$8,500
- (D) \$8,500 but less than \$9,500
- (E) \$9,500 or more

Data for Question 7 (4 points)

Smith and Jones are independent lives of the same age. They pay a single premium to purchase an annuity immediate of \$1,000, payable annually as long as at least one of them is alive.

Interest rate: 5%, compounded annually

$$l_x = Bc^{-x} \quad \text{for all } x > 0 \text{ with } c = 1.01$$

Question 7

In what range is the single premium?

- (A) Less than \$19,250
- (B) \$19,250 but less than \$19,750
- (C) \$19,750 but less than \$20,250
- (D) \$20,250 but less than \$20,750
- (E) \$20,750 or more

Data for Question 8 (3 points)

$$n|a_{\overline{2n}|} = 8.00407$$

$$n-1|a_{\overline{2n+1}|} = 8.63279$$

i = The annual rate of interest, compounded annually.

Question 8

In what range is i ?

- (A) Less than 4.80%
- (B) 4.80% but less than 4.90%
- (C) 4.90% but less than 5.00%
- (D) 5.00% but less than 5.10%
- (E) 5.10% or more

Data for Question 9 (3 points)

Independent lives Smith and Jones, both age 40, purchase an insurance policy with a death benefit of \$100,000 payable if and only if both die in the same year. The death benefit is payable at the end of the policy year.

Interest rate: 5%, compounded annually

$$l_x = 100 - x$$

Question 9

In what range is the single premium for this policy?

- (A) Less than \$500
- (B) \$500 but less than \$520
- (C) \$520 but less than \$540
- (D) \$540 but less than \$560
- (E) \$560 or more

Data for Question 10 (4 points)

Date of loan: 1/1/2005.

Terms of loan repayment: 20 annual payments beginning 12/31/2005. The payment is X in the first 10 years, and 50% of X in the second 10 years.

Interest rate: 5%, compounded annually.

Y is the ratio of principal repaid in the 10th payment to principal repaid in the 11th payment.

Question 10

In what range is Y ?

- (A) Less than 2.40
- (B) 2.40 but less than 2.46
- (C) 2.46 but less than 2.52
- (D) 2.52 but less than 2.58
- (E) 2.58 or more

Data for Question 11 (3 points)

Data for a participant in a defined contribution plan:

| | |
|----------------------|--|
| Age at 1/1/2005: | 40 |
| 2005 salary: | \$50,000 |
| Investment returns: | 5%, compounded annually. |
| Salary increases: | 4% annual increases over the salary in the prior year. |
| Contributions: | 6% of annual salary at the end of each year. |
| Balance at 1/1/2005: | \$0 |

X is the account balance as of 1/1/2030.

Question 11

In what range is X?

- (A) Less than \$192,000
- (B) \$192,000 but less than \$201,600
- (C) \$201,600 but less than \$211,200
- (D) \$211,200 but less than \$220,800
- (E) \$220,800 or more

Data for Question 12 (4 points)

Data from a double-decrement table:

$$\begin{array}{lll} l_{63}^{(\tau)} = 500 & l_{66}^{(\tau)} = 0 & \\ q_{63}^{(1)} = 0.050 & q_{63}^{(2)} = 0.500 & \\ {}_1q_{63}^{(1)} = 0.070 & {}_2q_{63}^{(1)} = 0.042 & {}_2q_{63}^{(2)} = 0.600 \end{array}$$

Question 12

In what range is $d_{65}^{(2)}$?

- (A) Less than 97
- (B) 97 but less than 103
- (C) 103 but less than 109
- (D) 109 but less than 115
- (E) 115 or more

Data for Question 13 (3 points)

$$\mu_x = 0.1, \text{ for } x > 0$$

P = The probability that two independent lives age 30 and 50 will die within 10 years of each other.

Question 13

What is P?

(A) $0.1e^{-1}$

(B) $0.5e^{-1}$

(C) e^{-1}

(D) $0.5(1 - e^{-1})$

(E) $1 - e^{-1}$

Data for Question 14 (3 points)

$$q_{60} = 0.020$$

$$q_{61} = 0.022$$

Deaths are uniformly distributed over each year of age.

$$X = \overset{\circ}{e}_{60:\overline{1.5}|}$$

Question 14

In what range is X ?

- (A) Less than 1.450
- (B) 1.450 but less than 1.460
- (C) 1.460 but less than 1.470
- (D) 1.470 but less than 1.480
- (E) 1.480 or more

Data for Question 15 (3 points)

Smith (age 40) is given the following actuarially equivalent payment options:

- (1) A lump sum payment of \$10,000; or
- (2) An annual payment of X at the beginning of each year guaranteed for 10 years and continuing as long as Smith is alive.

Interest rate: 4%, compounded annually

$$A_{40} = 0.30$$

$$A_{50} = 0.35$$

$$A_{40:\overline{10}|}^1 = 0.09$$

Question 15

In what range is X ?

- (A) Less than \$539.00
- (B) \$539.00 but less than \$541.00
- (C) \$541.00 but less than \$543.00
- (D) \$543.00 but less than \$545.00
- (E) \$545.00 or more

Data for Question 16 (4 points)

For a defined benefit pension plan, you are given:

- (i) The retirement benefit is \$15 per month per year of service.
- (ii) Retirement is allowed beginning at age 63 with no benefit reduction.
- (iii) An employee currently age 45 was hired at age 40.

Selected values:

| Age | $q'_x^{(r)}$ | $\ddot{a}_x^{(12)}$ |
|-----|--------------|---------------------|
| 63 | 0.2 | 10.0 |
| 64 | 0.3 | 9.5 |
| 65 | 1.0 | 9.0 |

Employees retire only on their birthdays, and there are no decrements other than retirement.

Interest rate: 5%, compounded annually

X is the present value at age 45 of the projected retirement benefit for this employee.

Question 16

In what range is X?

- (A) Less than \$13,500
- (B) \$13,500 but less than \$14,500
- (C) \$14,500 but less than \$15,500
- (D) \$15,500 but less than \$16,500
- (E) \$16,500 or more

Data for Question 17 (4 points)

Smith retires on 1/1/2005 and receives his retirement benefit as a monthly annuity, payable at the end of each month for a period certain of 20 years.

The benefit for the first year is \$2,000 per month. This monthly benefit is increased at the beginning of each year to be 5% larger than the monthly payment in the prior year.

X is the present value on 1/1/2005 of the retirement benefit at a nominal interest rate of 6%, convertible monthly.

Question 17

In what range is X?

- (A) Less than \$405,000
- (B) \$405,000 but less than \$410,000
- (C) \$410,000 but less than \$415,000
- (D) \$415,000 but less than \$420,000
- (E) \$420,000 or more

Data for Question 18 (3 points)

\$150 is invested in fund A at the end of each year for 20 years at an annual effective rate of 15%.

Interest from fund A is payable annually and is immediately invested in fund B at an annual effective rate of 11%.

X is the value of fund B at the end of 20 years.

Question 18

In what range is X?

- (A) Less than \$9,100
- (B) \$9,100 but less than \$9,500
- (C) \$9,500 but less than \$9,900
- (D) \$9,900 but less than \$10,300
- (E) \$10,300 or more

Data for Question 19 (2 points)

You are given the following cash flows:

\$10,000 payable 12/31/2005

\$20,000 payable 12/31/2007

\$15,000 payable 12/31/2008

X is the duration (Macaulay duration) as of 1/1/2005 of the above cash flows, measured at 6% interest, compounded annually.

Question 19

In what range is X?

- (A) Less than 2.72
- (B) 2.72 but less than 2.76
- (C) 2.76 but less than 2.80
- (D) 2.80 but less than 2.84
- (E) 2.84 or more

Data for Question 20 (4 points)

Smith and Brown are independent lives.

Smith is age x as of 1/1/2005.

Brown is exactly one year older than Smith.

The probability that neither Smith nor Brown dies during 2005 is equal to 150% of the probability that Brown dies during 2005.

Mortality data:

$$\overset{\circ}{e}_x = 2.0$$

$$\overset{\circ}{e}_{x+1} = 1.5$$

Deaths are uniformly distributed throughout each year of age.

$$Y = e_{x+2}$$

Question 20

In what range is Y ?

- (A) Less than 0.25
- (B) 0.25 but less than 0.45
- (C) 0.45 but less than 0.65
- (D) 0.65 but less than 0.85
- (E) 0.85 or more

Data for Question 21 (4 points)

Data on a bond:

Face amount of bond: \$1,000.

Terms of redemption: \$1,000 is redeemable after 10 years.

Coupon rate: 5% per year, payable semi-annually.

X = Premium if yield to maturity is 4%, compounded annually

Y = Discount if yield to maturity is 6%, compounded annually

Question 21

In what range is X minus Y?

- (A) Less than \$7
- (B) \$7 but less than \$10
- (C) \$10 but less than \$13
- (D) \$13 but less than \$16
- (E) \$16 or more

Data for Question 22 (2 points)

Consider the following statements:

- I. The longer a bond's maturity, the greater the bond's price sensitivity to changes in interest rates.
- II. The higher the coupon rate, the greater the bond's price sensitivity to changes in interest rates.
- III. The earlier a bond is callable at par, the greater the bond's price sensitivity to changes in interest rates.

Question 22

Which, if any, of the above statements is (are) true?

- (A) I and II only
- (B) I and III only
- (C) II and III only
- (D) I, II, and III
- (E) The correct answer is not given by (A), (B), (C), or (D) above.

Data for Question 23 (3 points)

$${}_{15}P_{25} = 0.8108$$

$${}_{15|15}q_{25} = 0.2027$$

$${}_{10}q_{55} = 0.2222$$

$$X = {}_{30|10}q_{25}$$

Question 23

In what range is X?

- (A) Less than 0.015
- (B) 0.015 but less than 0.065
- (C) 0.065 but less than 0.115
- (D) 0.115 but less than 0.165
- (E) 0.165 or more

Data for Question 24 (4 points)

Smith (age 55) is entitled to an annual payment of X at the beginning of each year guaranteed for 10 years and continuing as long as Smith is alive. Instead, Smith elects an actuarially equivalent annuity that pays the following as long as Smith is alive:

- (1) \$10,000 at the beginning of each year for the first 5 years,
- (2) \$7,500 at the beginning of each year for the next 5 years; and
- (3) \$5,000 at the beginning of each year thereafter.

Interest rate: 7%, compounded annually.

Selected actuarial factors:

$$\ddot{a}_{55} = 11.2751$$

$$\ddot{a}_{60} = 10.2758$$

$$\ddot{a}_{65} = 9.1301$$

$$\ddot{a}_{55:\overline{5}|} = 4.3122$$

$$\ddot{a}_{60:\overline{5}|} = 4.2707$$

Question 24

In what range is X ?

- (A) Less than \$7,375
- (B) \$7,375 but less than \$7,425
- (C) \$7,425 but less than \$7,475
- (D) \$7,475 but less than \$7,525
- (E) \$7,525 or more

Data for Question 25 (3 points)

Loan Amount: \$10,000

Payment Terms: Two payments:
End of year 1: X
End of year 2: $1.1X$

Force of Interest: $0.06 + 0.01t$, for $t \leq 2$

Question 25

In what range is X ?

- (A) Less than \$5,210
- (B) \$5,210 but less than \$5,280
- (C) \$5,280 but less than \$5,350
- (D) \$5,350 but less than \$5,420
- (E) \$5,420 or more

Data for Question 26 (4 points)

Participant date of birth: 1/1/1965

Disability benefit: \$25,000 payable at the end of each year beginning in the year of disability and continuing for life except that payments stop upon attainment of age 65.

Interest rate: 7%, compounded annually.

Recovery rate ($q_x^{(rec)}$): 3% per year after the year of disability.

Mortality rate ($q_x^{(d)}$): 8% per year after year of disability.
No mortality before disability.

Death and recovery are assumed to occur at the end of each year after any payments are made.

$$q_{40}^{(i)} = 0.005$$

Y is the term cost as of 1/1/2005

Question 26

In what range is Y?

- (A) Less than \$697.50
- (B) \$697.50 but less than \$722.50
- (C) \$722.50 but less than \$747.50
- (D) \$747.50 but less than \$772.50
- (E) \$772.50 or more

Data for Question 27 (5 points)

$$q_{[40]+s}^{(w)} = q_{[40]}^{(w)} - 0.02s; s = 0 \text{ or } 1$$

$$q_x^{(w)} = 0.10 - 0.003(x - 40)$$

All rates of withdrawal are less than 0.50.

Number of participants as of 1/1/2005:

| <u>Years of Service</u> | <u>Age 40</u> | <u>Age 41</u> |
|-------------------------|---------------|---------------|
| 0 | 200 | 0 |
| 1 | 0 | 150 |
| 2 or more | 900 | 0 |

120 of these individuals are expected to terminate employment in 2006.

Question 27

In what range is $q_{[40]}^{(w)}$?

- (A) Less than 0.175
- (B) 0.175 but less than 0.185
- (C) 0.185 but less than 0.195
- (D) 0.195 but less than 0.205
- (E) 0.205 or more

Data for Question 28 (3 points)

Values from a double decrement table:

$$l_{40}^{(T)} = 500,000$$

$$l_{41}^{(T)} = 417,362$$

Associated value:

$$q'_{40}^{(1)} = 0.0934$$

Each decrement is uniformly distributed within each year of age in the associated single decrement table.

$$X = m_{40}^{(2)}$$

Question 28

In what range is X?

- (A) Less than 0.077
- (B) 0.077 but less than 0.079
- (C) 0.079 but less than 0.081
- (D) 0.081 but less than 0.083
- (E) 0.083 or more

Data for Question 29 (3 points)

Smith and Jones are independent lives.

Smith (age 65) and Jones (age 64) pay \$100,000 for an annuity with the following payments:

- (1) Annual payments of X at the beginning of each year and ending on the first death.
- (2) Annual payments of $(X - \$3,000)$ beginning at the end of the year of the first death and continuing for the lifetime of the survivor.

Interest rate: 7%, compounded annually.

Selected actuarial factors:

$$p_{64} = 0.991315$$

$$p_{65} = 0.990180$$

$$\ddot{a}_{65:64} = 9.1707$$

$$\ddot{a}_{65} = 10.8207$$

Question 29

In what range is X ?

- (A) Less than \$8,522
- (B) \$8,522 but less than \$8,622
- (C) \$8,622 but less than \$8,722
- (D) \$8,722 but less than \$8,822
- (E) \$8,822 or more

2005 EA-1 EXAMINATION
ANSWER KEY

| Question | Answer |
|----------|--------|
| 1 | B |
| 2 | B |
| 3 | B |
| 4 | D |
| 5 | C |
| 6 | C |
| 7 | A |
| 8 | A |
| 9 | C |
| 10 | C |
| 11 | D |
| 12 | E |
| 13 | E |
| 14 | D |
| 15 | A |
| 16 | D |
| 17 | D |
| 18 | A |
| 19 | D |
| 20 | C |
| 21 | E |
| 22 | E |
| 23 | D |
| 24 | A |
| 25 | C |
| 26 | A |
| 27 | E |
| 28 | D |
| 29 | C |